BLatt 4

Aufgabe 1:

$\hat{\delta}(z,\varepsilon) := z$ $\qquad$ $\hat{\delta}'(z,\varepsilon) := z$

$\hat{\delta}(z,aw) := \hat{\delta}(\delta(z,a),w)$ $\qquad$ $\hat{\delta}'(z,wa) := \delta(\hat{\delta}'(z,w),a)$

Induktion über die Länge des Eingabewortes x.

J-Anfg.: $|x| = 0 \Rightarrow x = \varepsilon$ $\quad \hat{\delta}(z,\varepsilon) = z = \hat{\delta}'(z,\varepsilon)$ und $|x| = 1 \Rightarrow x = a$ $\quad \hat{\delta}(z,a) = \delta(z,a) = \hat{\delta}'(z,a)$

J-Annahme: $|x| = n \Rightarrow \hat{\delta}(z,x) = \hat{\delta}'(z,x)$

J-Schritt: $|\tilde{x}| = n+1$, d.h. $\tilde{x} = \tilde{x}_1 \tilde{x}_2 \ldots \tilde{x}_n \tilde{x}_{n+1} = \tilde{x}_1 \tilde{v} = \tilde{x}_1 \tilde{w} x_{n+1}$ (Damit $|\tilde{v}| = n$ und somit J-Annahme anwendbar)

$$\hat{\delta}(z,\tilde{x}) = \hat{\delta}(\delta(z,\tilde{x}_1),\tilde{v}) \overset{\text{J-Ann.}}{=} \hat{\delta}'(\delta(z,\tilde{x}_1),\tilde{v}) = \hat{\delta}'(\delta(z,\tilde{x}_1),\tilde{w} x_{n+1})$$

$$= \delta(\hat{\delta}'(\delta(z,\tilde{x}_1),\tilde{w}),\tilde{x}_{n+1}) \overset{\text{J-Anfg.}^*}{=} \delta(\hat{\delta}(\delta(z,\tilde{x}_1),\tilde{w}),\tilde{x}_{n+1})$$

$$= \delta(\hat{\delta}(z,\tilde{x}_1\tilde{w}),\tilde{x}_{n+1}) \overset{\text{J-Ann.}}{=} \delta(\hat{\delta}'(z,\tilde{x}_1\tilde{w}),\tilde{x}_{n+1})$$

$$= \hat{\delta}'(z,\tilde{x}_1\tilde{w}\tilde{x}_{n+1})$$

$$= \hat{\delta}'(z,\tilde{x}) \qquad \square$$

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* Der Schritt $\hat{\delta}'(\delta(z,\tilde{x}_1),\tilde{w}) = \hat{\delta}(\delta(z,\tilde{x}_1),\tilde{w})$ ist vielleicht nicht sofort einsichtig, da ja $|\tilde{w}| = n-1 \neq n$ und somit hier die Annahme nicht verwendbar ist. Es gilt aber:

$$\hat{\delta}'(\delta(z,\tilde{x}_1),\tilde{w}) = \delta(\hat{\delta}'(\delta(z,\tilde{x}_1),\tilde{w}_1\ldots\tilde{w}_{n-3}),\tilde{w}_{n-2})$$

$$\vdots$$

$$= \delta(\delta(\ldots\delta(\hat{\delta}'(\delta(z,\tilde{x}_1),\tilde{w}_1),\tilde{w}_2)\ldots,\tilde{w}_{n-3}),\tilde{w}_{n-2})$$

$$\overset{\text{J-Anfg.}}{=} \delta(\delta(\ldots\delta(\delta(\delta(z,\tilde{x}_1),\tilde{w}_1),\tilde{w}_2)\ldots,\tilde{w}_{n-3}),\tilde{w}_{n-2})$$

$$= \hat{\delta}(\delta(\ldots\delta(\delta(\delta(z,\tilde{x}_1),\tilde{w}_1),\tilde{w}_2)\ldots,\tilde{w}_{n-3}),\tilde{w}_{n-2})$$

$$= \hat{\delta}(\delta(\ldots\delta(\delta(\delta(z,\tilde{x}_1),\tilde{w}_1),\tilde{w}_2)\ldots,\tilde{w}_{n-4}),\tilde{w}_{n-3}\tilde{w}_{n-2})$$

$$\vdots$$

$$= \hat{\delta}(\delta(\delta(z,\tilde{x}_1),\tilde{w}_1),\tilde{w}_2\ldots\tilde{w}_{n-2})$$

$$= \hat{\delta}(\delta(z,\tilde{x}_1),\tilde{w}_1\ldots\tilde{w}_{n-2})$$

$$= \hat{\delta}(\delta(z,\tilde{x}_1),\tilde{w})$$

Aufgabe 2:

$L(aa^*) = \{aa^n \mid n \geq 0\} = \{a^n \mid n > 0\} = \{a^n a \mid n \geq 0\} = L(a^* a)$

$L(a^*) = \{a^n \mid n \geq 0\} = \{\varepsilon\} \cup \{aa^n \mid n \geq 0\} = (\varepsilon \mid (aa^*)) = (\varepsilon \mid (aa^*))^*$ (wg. $\varepsilon^* = \varepsilon$; $(aa^*)^* = (aa^*)$)

$(a^* b^*)^* = (\varepsilon \mid aa^*b^* \mid a^*b^*b)^* = (\varepsilon \mid a \mid b)^* = (a \mid b)^*$

$L((ab)^* a) = \{(ab)^n a \mid n \geq 0\} = \{a(ba)^n \mid n \geq 0\} = L(a(ba)^*)$

Aufgabe 3:

$L_1 := \{x \mid \#_a(x) > 1 \Leftrightarrow \#_b(x) \leq 1\}$

Mehr als ein a's $\Rightarrow$ ≤ 1 b's

$aaa^* \mid baaa^* \mid aaa^*b$
$\mid abaa^* \mid aaba^*$
$\mid a^*baa \mid aa^*ba$

Mehr als ein b's $\Rightarrow$ ≤ 1 a's

$bbb^* \mid abbb^* \mid bbb^*a$
$\mid babb^* \mid bbab^*$
$\mid b^*abb \mid bb^*ab$

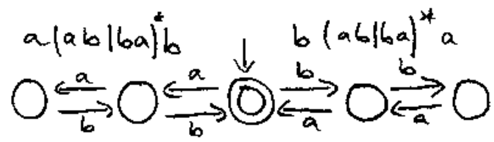
$L_1 = L((aaa^* \mid baaa^* \mid aaa^*b \mid abaa^* \mid aaba^* \mid a^*baa \mid aa^*ba \mid bbb^* \mid abbb^* \mid bbb^*a \mid babb^* \mid bbab^* \mid b^*abb \mid bb^*ab)$

$L_2 := \{x \mid 2 \mid \#_a(x) ; 2 \mid \#_b(x)\}$

$L_2 = L((aa \mid bb \mid (ab \mid ba)(aa \mid bb)^*(ab \mid ba))^*)$

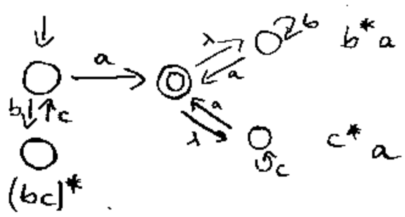
Aufgabe 4:

L_3:

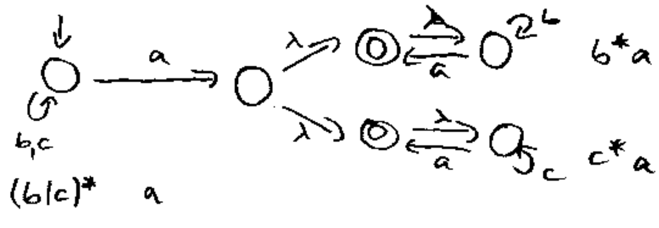


Sieht man es nicht direkt, steht auf S. 57f ein Verfahren...

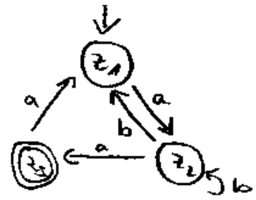
L_4:



L_5:



Aufgabe 5:



$T(M) = L(\alpha^3_{1,3})$

$\alpha^3_{1,3} = \alpha^2_{1,3} \mid \alpha^2_{1,3}(\alpha^2_{3,3})^* \alpha^2_{3,3}$

geht auch mit dem impliziten λ-Übergang $\delta(z_3,\lambda)=z_3$ und ist damit schon in $(\alpha^2_{3,3})^*$ enthalten!

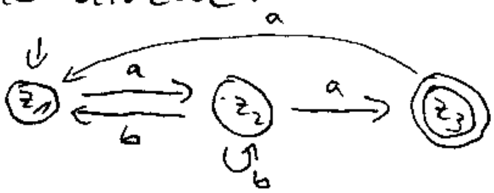
$= \alpha^2_{1,3}(\alpha^2_{3,3})^*$

$= (\alpha^1_{1,3} \mid \alpha^1_{1,2}(\alpha^1_{2,2})^* \alpha^1_{2,3})(\alpha^1_{3,3} \mid \alpha^1_{3,2}(\alpha^1_{2,2})^* \alpha^1_{2,3})^*$

$= a(b|ba)^* a \; (aa(b|ba)^* a)^*$

$= a(b|ba)^* a \, (aa\,(b|ba)^* a)^*$

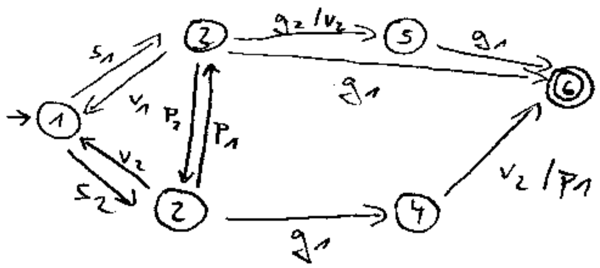
oder direkt:



$a(ba|b)^* a \, (a \, \underbrace{a(ba|b)^* a}_{a \,\hat{=}\, \text{das Ganze von vorne...}})^*$

Aufgabe 6:

a)



b) $T(M) = \alpha^6_{1,6} = \alpha^5_{1,6} \mid \alpha^5_{1,6}(\alpha^5_{6,6})^* \alpha^5_{6,6} = \alpha^5_{1,6} = \alpha^4_{1,6} \mid \alpha^4_{1,5}(\alpha^4_{5,5})^* \alpha^4_{5,6}$

$= \alpha^4_{1,6} \mid \alpha^4_{1,5}\, g_1$

Zwischenschritte:

1) $\alpha^4_{1,6} = \alpha^3_{1,6} \mid \alpha^3_{1,4}(\alpha^3_{4,4})^* \alpha^3_{4,6} = \alpha^3_{1,6} \mid \alpha^3_{1,4}(v_2|p_1)$

2) $\alpha^3_{1,6} = \alpha^2_{1,6} \mid \alpha^2_{1,3}(\alpha^2_{3,3})^* \alpha^2_{3,6} = \alpha^2_{1,3}(\alpha^2_{3,3})^* g_1 \overset{5)6)}{=} (s_1|s_2(v_2s_2)^*p_1)(v_1s_1|p_2(v_2s_2)^*p_1)^* g_1$

3) $\alpha^3_{1,4} = \alpha^2_{1,4} \mid \alpha^2_{1,3}(\alpha^2_{3,3})^* \alpha^2_{3,4} \overset{4)5)6)7)}{=} s_2(v_2s_2)^*g_1 \mid (s_1|s_2(v_2s_2)^*p_1)(v_1s_1|p_2(v_2s_2)^*p_1)^* p_2(v_2s_2)^* g_1$

4) $\alpha^2_{1,4} = \alpha^1_{1,4} \mid \alpha^1_{1,2}(\alpha^1_{2,2})^* \alpha^1_{2,4} = s_2(v_2s_2)^* g_1$

5) $\alpha^2_{1,3} = \alpha^1_{1,3} \mid \alpha^1_{1,2}(\alpha^1_{2,2})^* \alpha^1_{2,3} = s_1 \mid s_2(v_2s_2)^* p_1$

6) $\alpha^2_{3,3} = \alpha^1_{3,3} \mid \alpha^1_{3,2}(\alpha^1_{2,2})^* \alpha^1_{2,3} = \alpha^1_{3,3} \mid p_2(v_2s_2)^* p_1 \overset{!}{=} v_1s_1 \mid p_2(v_2s_2)^* p_1$

7) $\alpha^2_{3,4} = \alpha^1_{3,4} \mid \alpha^1_{3,2} (\alpha^1_{2,2})^* \alpha^1_{2,4} = p_2 (v_2 s_2)^* g_1$

8) $\alpha^1_{3,3} = \alpha^0_{3,3} \mid \alpha^0_{3,1} (\alpha^0_{1,1})^* \alpha^0_{1,3} = v_1 s_1$

Es Folgt:

$$\alpha^6_{1,6} = \alpha^4_{1,6} \mid \alpha^4_{1,5} g_1$$

$$= (s_1 \mid s_2 (v_2 s_2)^* p_1)(v_1 s_1 \mid p_2 (v_2 s_2)^* p_1)^* g_1 \mid \big(s_2 (v_2 s_2)^* g_1 \mid (s_1 \mid s_2 (v_2 s_2)^* p_1)$$

$$(v_1 s_1 \mid p_2 (v_2 s_2)^* p_1)^* p_2 (v_2 s_2)^* g_1 \big) (v_2 \mid p_1)$$